MATHS Quest MATHS B for Queensland YEAR 11 SECOND EDITION

WorkSHEET 2.2 Relations and functions

Name: ______________________________

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1 If $f(x)=3x^2-2x+1$ and $g(x)=3x-2$, find: 4

(a) $f(3)$

(b) $g(-2)$

(c) $f(3x-2)$

(d) $g(f(x))$

(e) $h(x)$, where $h(x)=f(x)+g(x)$

(f) $k(x)$, where $k(x)=f(x)-g(x)$

(g) $m(x)$, where $f(x)=g(x)-m(x)$

(a) $f(3)=3(3)^2-2(3)+1$
$=22$

(b) $g(-2)=3(-2)-2$
$=-8$

(c) $f(3x-2)=3(3x-2)^2-2(3x-2)+1$
$=3(9x^2-12x+4)-6x+4+1$
$=27x^2-36x+12-6x+4+1$
$=27x^2-42x+17$

(d) $g(f(x))=3(3x^2-2x+1)-2$
$=9x^2-6x+3-2$
$=9x^2-6x+1$

(e) $h(x)=f(x)+g(x)$

$=3x^2-2x+1\ +\ 3x-2$

$=3x^2+x-1$

(f) $k(x)=f(x)-g(x)$

$=3x^2-2x+1-\ (3x-2)$

$=3x^2-2x+1-3x+2$

$=3x^2-5x+3$

(g) $\mathrm{f}(x)=g(x)-m(x)$

so,

$m(x)=g(x)-f(x)$

$=3x-2\ -\ (3x^2-2x+1)$

$=3x-2-\ 3x^2+2x-1$

$\therefore\ m(x)=-3x^2+5x-3$

2	Determine the value of β, given that $f(x)$ cuts the x-axis where $x = 4$ and where $f(x) = x^2 + 2x + \beta$	Since the point (4,0) lies on the function, it must satisfy its equation. By sub; $$0 = 4^2 + 2 \times 4 + \beta$$ $$\beta = -24$$ You can justify this by graphing the function in Desmos: $y = x^2 + 2x - 24$
3	Consider an amalgamation of the last 2 questions: $$f(x) = x^2 + 2x + \beta$$ and $$g(x) = x^2 + 3x + 1$$ and $$m(x) = f(x) + g(x)$$ If m(x) cuts the x-axis where $x = 4$, determine the value of β and hence state $m(x)$.	$$m(x) = f(x) + g(x)$$ $$= x^2 + 2x + \beta + x^2 + 3x + 1$$ $$= 2x^2 + 5x + 1 + \beta$$ Since the point (4,0) lies on the function, it must satisfy its equation. By sub; $$0 = 2 \times 4^2 + 5 \times 4 + 1 + \beta$$ $$\beta = -53$$ Hence, $$\therefore \quad m(x) = 2x^2 + 5x - 52$$ You can justify this by graphing the function in Desmos: $m(x) = 2x^2 + 5x - 52$
4	Determine the type of relation of: (a) $y = 3x^2 - 2x + 1$ (b) $y = 3x - 2$ (c) $x^2 + y^2 = 36$ (d) $x = y^2$	4 (a) $y = 3x^2 - 2x + 1$ (a parabola) many-to-one relation (b) $y = 3x - 2$ (straight line) one-to-one-relation (c) $x^2 + y^2 = 36$ (circle) many-to-many relation (d) $x = y^2$ (a parabola, symmetrical about x-axis) one-to-many relation

5	Which of the relations in the last question are functions?	Only parts (a) and (b) are functions. (Only one-to-one and many-to-one relations are functions.) Justification: a function is where for every value of x, there is only 1 value of y.	2
6	Which of the following graphs are one-to-one functions? (use your calculator to graph) (a) $y = 3\sin x$ (b) $y = x^2 - 1$ (c) $y = 1 - 3x$ (d) $y = \sqrt{1 + x^2}$ (e) $y = -\sqrt{2x - 1}$	Only (c) and (e) are one-to-one functions. [(a), (b) and (d) correspond to many-to-one functions, as a horizontal line on each graph will pass through more than one *x*-value.]	2
7	If $f(x) = \begin{cases} 2 - x, & x \le 2 \\ 4 - x^2, & x > 2 \end{cases}$ (a) state the range of *f* (b) find *f*(1), *f*(2) and *f*(3).	(a) As the two separate parts of the graph connect, the range is *R*. (b) $f(1) = 2 - 1 = 1$ $f(2) = 2 - 2 = 0$ $f(3) = 4 - 3^2 = -5$	4

8 State the domain, range and if the following are functions:
(Use your calculator to graph)

a) $y = 3\sin x$

b) $y = x^2 - 1$

c) $y = 1 - 3x$

d) $y = \sqrt{1 + x^2}$

Plot this next one on Desmos,

e) $(y-2)^2 = x + 1$

Which of the following graphs are one-to-one functions? 3

a) $y = 3\sin x$

$$-\infty < x < \infty$$
$$-3 \leq y \leq 3$$
Function.

b) $y = x^2 - 1$

$$-\infty < x < \infty$$
$$-1 \leq y \leq \infty$$
Function.

c) $y = 1 - 3x$

$$-\infty < x < \infty$$
$$-\infty \leq y \leq \infty$$
Function.

d) $y = \sqrt{1 + x^2}$

$$-\infty < x < \infty$$
$$1 \leq y < \infty$$
Function.

e) $y = -\sqrt{2x - 1}$

$$-1 \leq x < \infty$$
$$-\infty < y < \infty$$
NOT a Function, because it fails the VLT!

9 State the implied domain restriction:

$$y = \frac{1}{x}$$

Think :

$$\frac{1}{0} \to can't\ happen$$

$$\therefore -\infty < x < \infty, \qquad but\ x \neq 0$$

10 State the implied domain restriction:

$$y = \frac{1}{x-2} + 3$$

Think :

$$\frac{1}{0} \to can't\ happen$$

$$\therefore -\infty < x < \infty, \qquad but\ x \neq 2$$

11 State the implied domain restriction:

$$y = \sqrt{x}$$

Think :

$$\sqrt{-ve} \rightarrow can't\ happen$$

$$\therefore 0 \leq x < \infty$$

12 State the implied domain restriction:

$$y = \sqrt{x - 1}$$

Think :

$$\sqrt{-ve} \rightarrow can't\ happen$$

Clearly $x = 1$ is important

$$\therefore 1 \leq x < \infty$$

13 State the implied domain restriction:

$$y = \sqrt{x^2 - 9}$$

Think :

$$\sqrt{-ve} \rightarrow can't\ happen$$

Clearly $x = 3\ or\ maybe - 3$ is important

Needs further investigation … check:

-4	-3	-2	-1	0	1	2	3	4
✓	✓	×	×	×	×	×	✓	✓

Hence;

$$\therefore -3 \geq x \geq 3$$

14 State the implied domain restriction:

$$y = \sqrt{9 - x^2}$$

Think :

$$\sqrt{-ve} \rightarrow can't\ happen$$

Clearly $x = 3\ or\ maybe - 3$ is important

Needs further investigation … check:

-4	-3	-2	-1	0	1	2	3	4
×	✓	✓	✓	✓	✓	✓	✓	×

Hence;

$$\therefore -3 \leq x \leq 3$$

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Implied domains … !

Do Question 18 on page 53

16	State the translations of: a) $y = 2(x-1)^2 + 4$ b) $y = 2(x-1)^3 + 4$ c) $y = 2(x-1)^4 + 4$ d) $y = 2\sqrt{x-1} + 4$ e) $y = \frac{2}{x-1} + 4$	a, b, c, d Dilation increased to 2 … makes graph Steeper Graph moved 1 unit to the Right Graph moved 4 units Up e) Dilation increased to 2 … makes graph flatter Graph moved 1 unit to the Right Graph moved 4 units Up
17	If $f(x) = x + 1$, determine $f(2)$.	$f(2) = 2 + 1 = 3$
18	If $f(x) = x + 1$, determine $f(x+1)$.	$f(x+1) = x + 1 + 1 = x + 2$
19	If $f(x) = x + 1$, determine the value of x such that $f(2) = f(x+1)$.	$f(2) = f(x+1)$ $3 = x + 2$ $x = 1$
20	If $f(x) = x^2 + 2x + 1$, determine $f^{-1}(x)$. (recall that $f^{-1}(x)$ means find the inverse function)	Set $y = x^2 + 2x + 1$ To find inverse, swap x *and* y $x = y^2 + 2y + 1$ $x = (y+1)^2$ $\pm\sqrt{x} = y + 1$ $y = \pm\sqrt{x} - 1$ $\therefore\ f^{-1}(x) = \pm\sqrt{x} - 1$

21	David rides his bike at a constant speed of 20 km/h for 3 hours, stops for 1 hour to rest, and then travels for another 2 hours at a constant speed of 25 km/h to reach his destination. (a) Construct a function that describes the distance, d (km), that David has travelled at time t (hours).	(a) For the first 3 hours, the distance travelled, $d = 20t$. Between the 3rd and 4th hours, David does not travel, therefore the distance travelled is the same as that travelled when $t = 3$; that is, $d = 20 \times 3 = 60$ km Between the 4th and 6th hours, the distance travelled, $d = 25t - 40$ (allows $d(t) = 60$, when $t = 4$). $d(t) = \begin{cases} 20t & 0 \le t \le 3 \\ 60 & 3 < t \le 4 \\ 25t - 40 & 4 < t \le 6 \end{cases}$	5
	(b) State the domain and range of this function.	(b) Domain: [0, 6] When $t = 0$, $d = 0$. When $t = 6$, $d = 110$. Range: [0, 110]	
22	Is the factor and remainder theorem in the exam?	YES. Make sure you do revision in Chapter 3 and 5 of the textbook!	